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RESEARCH ARTICLE

EDGE GEODETIC SPANNING GRAPH AND EDGE GEODETIC EDGE MINIMAL NUMBER

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ABSTRACT

Let G= (V, E) be a connected graph with p vertices V (G) and q edges E(G). Let the edge geodetic number of G is $g_1(G)$*. A subset M of E said to be edge geodetic edge minimal set of G if the edge geodetic number of spanning sub graph G-M equals the edge geodetic number of G. That is* $g_1(G\text{-}M)=g_1(G)$*. The maximum cardinality of M is called the edge geodetic edge minimal number, is denoted by* $g_{E_0}(G)$ *and the spanning sub graph G-M is called edge geodetic spanning graph of G, denoted by* G_1 *and* $E(G_1)$ *is denoted by* q_1*. Edge geodetic edge minimal number of some standard graphs are determined and we have the realization result for each pair of integers a, b and* $3 \le a \le b$*, there exist a connected graph G such that* $g_{E_0} = b$*,* $g_1 = a$ *and* $q=a+b$*.*

1. Introduction

By a graph *G* = (*V*, *E*), we mean a finite undirected graph without loops or multiple edges. The order and size of *G* are denoted by *p* and *q* respectively. For basic graph theoretic terminology we refer to Harary [3,7]. The distance *d*(*u*,*v*) between two vertices *u* and *v* in a connected graph *G* is the length of a shortest *u*-*v* path in *G*. An *diameter*, *diam G* of *G*. An *edge geodetic set* of *G* is a set $S \subseteq V$ such that every edge of *G* is contained in a geodesic joining some pair of vertices in *S*. The *edge geodetic number* $g_1(G)$ of *G* is the minimum order of its edge geodetic sets and any edge geodetic set of order $g_1(G)$ is an *edge geodetic basis* of *G* or a g_1-set of *G*. The edge geodetic number of a graph *G* is studied in [1, 8].

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$N(v)$ = { $u \in V(G)$: $uv \in E(G)$} is called the neighborhood of the vertex v in G. For any set S of vertices of G, the *induced sub graph* <S> is the maximal sub graph of G with vertex set S. If a sub graph of G has the same vertex set as G, then it is a *spanning sub graph* of G. A vertex v is a *simplified vertex* of a graph G if <$N(v)$ > is complete. A simplex of a graph G is a sub graph of G which is a complete graph. If $e = \{u, v\}$ is an edge of a graph G with $d(u) = 1$ and $d(v) > 1$, then we call e a pendent edge, u a leaf and v a support vertex and $L(G)$ be the set of all leaves of a graph G.

Theorem1.1.[8] Let G=(p,q) be a connected graph with exactly one vertex of degree p-1,then g_1 (G)=p-1.

Theorem 1.2 [8] Let G=(p,q) be a connected graph with more than one vertex of degree p-1, then g_1 (G)=p.

Theorem 1.3 [8] For the cycle C_p (p≥3),

$$g_1(C_p) = \begin{cases} 2 \text{ if n is even} \\ 3 \text{ if n is odd} \end{cases}$$

Edge Geodetic spanning Graph and Edge Geodetic Edge Minimal number

Definition 2.1 Let G=(V,E) be a connected graph with p vertices V(G) and q edges E(G) and edge geodetic number g_1 (G). A subset M of E is said to be an edge geodetic edge minimal set of G if the edge geodetic number of spanning sub graph G-M equals the edge geodetic number of G. That is g_1 (G-M)=g_1 (G). The maximum cardinality of M is called the edge geodetic edge minimal number, is denoted by g_{E_0} (G) and the spanning sub graph G-M is called edge geodetic spanning graph, denoted by $G_1$and E(G_1) is denoted by q_1.

Example:2.2 For the graph given in Figure 2.1, S={ v_1, v_3, v_4, v_6 } is an edge geodetic set and hence the

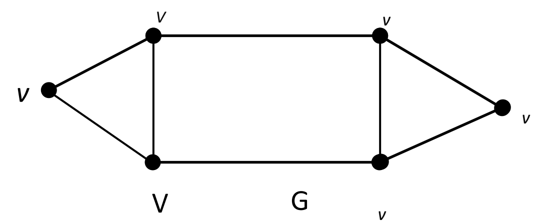

Figure 2.1

edge geodetic number is 4. Let M={ v_1 v_2, v_2 v_3, v_3v_4 } . Then the graph G_1 is given in Figure 2.2 Since the edge geodetic number of G_1 is 4, M is an edge minimal set of G and hence g_{E_0}≥ 3. It is easily verified that there is no edge minimal set of cardinality greater than 3. Therefore g_{E_0}= 3.

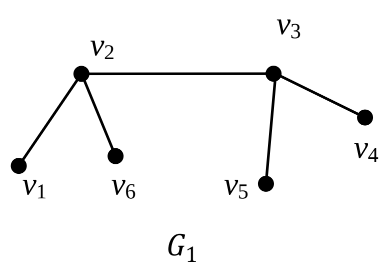

Figure 2.2

Remark 2.3 There can be more than one edge geodetic edge minimal set for a graph. For the graph in Figure 2.1 M_1={ v_1 v_2, v_2 v_3, v_3v_4 } M_2={ v_1 v_6, v_6 v_5, v_5 v_4 } are the two edge geodetic edge minimal set.

Theorem2.4. No bridge edge belongs to any edge geodetic edge minimal set

Proof: Suppose $e \in M$ is a bridge edge of G ,then <E-M> will be a disconnected graph, which is a contradiction. Hence no bridge edge belongs to any edge geodetic edge minimal set of G.

Corollary 2.5 No pendent edge belongs to edge geodetic edge minimal set.
Proof: The proof follows the Theorem 2.4.

Corollary 2.6 For any tree T, $g_{E_0}(T) = 0$.
Proof: The proof follows the Theorem 2.4 and Corollary 2.5.

Theorem: *2.7.* For any connected graph $0 \le g_{E_0}(G) \le q-\ell$, where ℓ is the number of bridge edges.

Proof: The proof follows the **Theorem 2.4.**

Theorem2.8. For any connected graph G, $0 \le g_{E_0}(G) \le q-1$.

Proof: Any edge geodetic set needs at least two vertices, minimum one edge and hence $g_{E_0}(G) \le q-1$. Also set of all vertices of G form an edge geodetic basis, maximum q edges and hence $g_{E_0}(G) \ge 0$.

Remark 2.9 The equality in the Theorem2.8 hold for K_2 and C_n. g_{E_0} $(K_2)=0$ and for even cycle C_n , g_{E_0} (C_n) =q-1 . Also the in equality in the Theorem 2.8 is strict. For the graph G given Figure 2.3, S={ v_1, v_2, v_4 } is an edge geodetic basis and hence the edge geodetic number g_1(G)=3, M= { $v_1 v_5, v_2 v_3$ } is an edge geodetic edge minimal set of G and hence $g_{E_0}(G) = 2$ .Thus $0 < g_{E_0}(G) < q-1$.

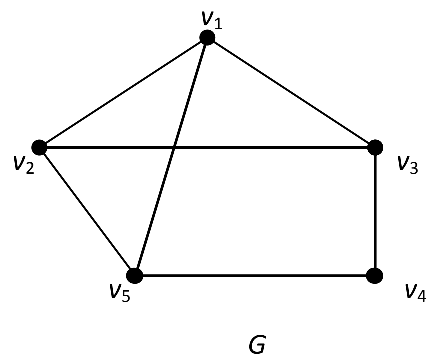

Figure 2.3

Theorem2.10 For any cycle

$$g_{E_0}(C_p) = \begin{cases} 1 & \text{if p is even} \\ 0 & \text{if p is odd} \end{cases}$$

Proof:
Case (i) p is even.
Let e be an edge of C_n then $C_n - e$ is a path so that $g_{E_0}(C_n - e) \ge 1$ .Since G-M is disconnected for $|M| \ge 2$, by the Theorem 1.3, it follows that $g_{E_0}(C_n)$ =1.
Case (ii) p is odd. Let$\{e_1, e_2, e_3, \ldots. e_p\}$ be the edges of C_p and $\{u_1, u_2, \ldots, u_p\}$ be the vertices . Take $e_i = u_i u_{i+1}$ then G-$\{e_i\}$ $=u_1, u_2, \ldots, u_{i-1}, u_{i+2 \ldots\ldots\ldots} u_p$ form a path. For path the edge geodetic number is 2 only. Also G-$\{e_{i,}\ e_j\}$ become a disconnected graph. Hence removal of any number of edges from C_n, the resultant graph is disconnected, cannot give the edge geodetic number 3.Hence $g_{E_0}(C_p) = 0$ if n is odd.

Theorem2.11. If G is a Simple graph of order p with exactly one vertex of degree p-1 then edge geodetic spanning graph is a Star and $g_{E_0}(G) + g_1(G)$=q.

Proof: Let G be a simple graph of order n with exactly one vertex of degree p-1. Let $\{u_1, u_2, \ldots, u_p\}$ be the vertices of G. Without loss of generality assume that the degree of u_i is p-1 Fix the vertex u_i which has the degree p-1 and collect all the edges from remaining vertices $\{u_1, u_2, \ldots u_{i-1}, u_{i+1} \ldots, u_p\}$ which are not adjacent with u_i. Hence the resultant graph is a star of order p. Removal of any edges from star form a disconnected graph. Also the edge geodetic number of star is p-1. Hence star is the edge geodetic spanning graph of order p and q_1=p-1, which is equal to $g_1(G)$. Also $g_{E_0}(G)$= q-q$_1$=q- $g_1(G)$ and so that $g_{E_0}(G)+ g_1(G)$=q.

Theorem 2.12 If G is a Simple graph of order n with exactly one vertex of degree p-1 then edge geodetic edge minimal number $g_{E_0}(G) = q - \Delta(G)$.

Proof: The proof follows from the Theorem 2.11.

Theorem2.13 for any Wheel $W_{1,p}$ $g_{E_0}(G)$=p.

Proof: Let G be Wheel graph .Wheel has exactly one vertex of degree p and remaining p vertex of degree 3. Hence the total number of edges is q=$\frac{p+3p}{2}$=2p. Since exactly one vertex of degree p, by Theorem 2.11 the edge geodetic spanning graph of G is a star of order p+1 and has edges p. Hence $g_{E_0}(G)$=2p-p. Hence $g_{E_0}(G)$= p.

Theorem 2.14 If G be a simple graph with more than one vertex of degree n-1 then the edge geodetic edge minimal number $g_{E_0}(G)$=q-2$\Delta(G) + 1$.

Proof: Let G be a simple graph with at least two vertices of degree p-1. Let $\{u_1, u_2, \ldots, u_p\}$ be the vertices of G. Without loss of generality assume that degree of u_i and u_j is p-1. Fix the vertices u_i and u_j .collect all the edges of $\{u_1, u_2, \ldots u_{i-1}, u_{i+1} \ldots u_{j-1}, u_{j+1} \ldots u_p\}$ which are not adjacent to u_i and u_j .Thus the new graph has two vertices of degree p-1 and remaining p-2 vertices of degree 2. Since G has more than one vertex of degree p-1, edge geodetic number $g_1(G) = p$. Hence the resultant graph has edge geodetic number p. So the graph is edge geodetic spanning graph of G. If removal of any more edge from G than the above condition degree of u_i or u_j will reduce to n-2 or less and hence has edge geodetic number less than n and cannot be the edge geodetic spanning graph of G so that q_1= $\frac{2\times(p-1)+(p-2)\times 2}{2}$

=2p-3.Therefore $g_{E_0}(G)$=q-(2p-3)
= q-2(p-1)+1
= q-2$\Delta(G) + 1$.

Theorem 2.15 For any complete graph K_n, $g_{E_0}(K_n)$=$\frac{(p-2)(p-3)}{2}$.

Proof: For the complete graph of order p has $\frac{p(p-1)}{2}$ *edges* and $\Delta(G)$= p-1, the proof follows from the Theorem 2.14.

Therorem2.16

$$g_{E_0}(K_{mn}) = \begin{cases} 0, & if \quad m = 1 \\ (m-1)^2, & if \quad m = n \end{cases}$$

Proof:

Casei: If m=1, the proof follows the Theorem 2.11

Case ii: if m=n

Let $U = \{u_1, u_2, \ldots, u_m\}$ and $W = \{w_1, w_2, \ldots, w_n\}$ be a bipartition of *G*. Let $\{e_{11}, e_{12}, e_{13}, \ldots e_{mm}\}$ such that e_{1i} (i=1,2,…..m) is incident with u_1 and w_i , e_{2i} (i=1,2,…..m) is incident with u_2 and w_i an so on. Collect all the edges except

e_{11}, e_{12} from u_1, e_{22} from u_2, e_{32}, e_{33}, e_{34} from u_3, e_{44} from u_4, e_{54}, e_{55}, e_{56} from u_5, e_{66} from e_6….. , e_{mm} from u_m if m is odd and e_{mm} from u_m if m is even. Hence the resultant graph be a tree such that $u_1, u_3, \ldots, u_{m-1}, w_2, w_4, \ldots, w_m$ are end vertices if m is odd and $u_1, u_3, \ldots, u_m, w_2, w_4, \ldots, w_{m-1}$ are end vertices if m is even .Also the resulting graph is a tree with m end vertices and hence the edge geodetic number is m which is same as the edge geodetic number of $G=K_{mm}$. Removal of any more edges from the resulting graph, the graph becomes disconnected and hence no spanning subgraph graph exist with same edge geodetic number m .Hence the tree is the edge geodetic spanning graph of G with m-2 vertices of degree 3, m vertices of degree one and 2 vertices of degree 2.Therefore the edge geodetic spanning graph of G has $\frac{3(m-2)+m+4}{2}$=2m-1 edges. Also the edge geodetic edge minimal number $g_{E_0}=m^2-(2m-1)$.That is $g_{E_0}=(m-1)^2$ if m=n.

Theorem:2.17 Let G be a connected graph with exactly one vertex of degree p-1 then $g_{E_0}=\sum_{i=1}^{p-1}\frac{d(v_i)}{2}-\frac{p-1}{2}$.

Proof: Let $\{v_1, v_2, \ldots, v_p\}$ be the vertices of G with exactly one vertex v_p(say)of degree p-1 .By the Theorem 2.12 ,edge geodetic edge minimal set contains E (G)-number edges in a star with central vertex v_p is $[d(v_1)-1+d(v_2)-1+\ldots\ldots+d(v_{p-1})-1]/2$ $=\sum_{1}^{p-1}\frac{d(v_i)}{2}-\frac{p-1}{2}$.

Therorem2.18. Let $\{x, y, v_1, v_2, \ldots, v_{p-2}\}$be the vertices of G, where x and y are universal vertices of G. Then $g_{E_0}=\sum_{i=1}^{p-2}\frac{d(v_i)-2(p-2)}{2}$.

Proof: Let G be the connected graph with more than one vertex of degree p-1. Let $\{x, y, v_1, v_2, \ldots, v_{p-2}\}$ be the vertices of G. Then the smallest graph with exactly two vertices of degree p-1 has exactly two vertices of degree p-1 and p-2 vertices of degree 2. Also the edge geodetic number of the resultant graph is p. Hence the edge geodetic edge minimal set contains $=(d(v_1)-2+d(v_2)-2+\cdots\ldots\ldots\ldots.+d(v_{p-2})-2)/2$. Hence $g_{E_0}=\sum_{i=1}^{p}\frac{d(v_i)-2(p-2)}{2}$.

Theorem 2.19 Let G=(p,q) be a connected graph with $g_1(G)$=p then q≥2p-3,(p≥2).

Proof: We prove this theorem by induction on p (p≥2). Suppose p=2, then $G=K_2$, the result is trivially true. Assume that for any connected graph with k vertices and $g_1(G)$=k has q≥2k-3 edges. Let G' be a connected graph obtained from G by adding a new vertex v .Let $\{v_1, v_2, \ldots, v_k\}$be the vertices of G and S be the edge geodetic basis of G. Since $g_1(G)$=k then S is the unique edge geodetic basis of G and either G has more than one vertex of degree k-1 or G has no vertex of degree k-1.

Case(i). Suppose G has more than one vertex of degree k-1.Let $d(v_i)=d(v_j)$=k-1.Add new vertex v to G such that v is adjacent to both v_i, v_j. Hence $d(v_i)=d(v_j)$=k so that $g_1(G')$=k+1.Also q≥2k-3+2=2(k+1)-3.Hence the result is true.

Case(ii). Suppose no vertex of G has degree p-1.Let S' be an edge geodetic basis of G'. Add a new vertex v such that v is adjacent to any two adjacent vertices v_i, v_j,(i≠j), (1≤I,j≤k).Hence v becomes an extreme vertex. Clearly $v_1, v_2, \ldots\ldots v_{i-1}, v_{i+1}, \ldots v_{j-1}, v_{j+1} \ldots v_k, v \in S'$.To prove $g_1(G')$=k+1 Suppose both v_i, v_j,(i≠j),

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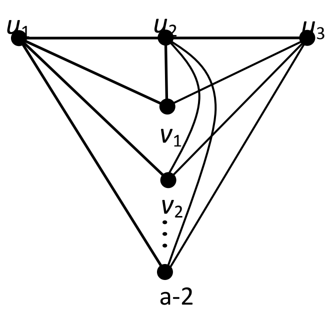
$(1 \le I, j \le k)$ does not belongs to S', then the edge v_iv_j cannot lies on geodesic joining of pair of vertices of S'. Hence anyone v_i,v_j or both v_i,v_j belongs to S'. Suppose v_i does not belongs to S', then the edge v_iv_j , v_iv cannot lies on the geodesic joining of pair of vertices of S'. Similarly if v_j does not belongs to S' then the edges v_i v_j, v_j,v cannot lies on the geodesic joining of pair of vertices of S'. Hence both v_i ,v_j belongs to S' and so that $g_1(G')=k+1$. Also $q \ge 2k-3+2=2(k+1)-3$.Hence $q \ge 2p-3,(p \ge 2)$.

Theorem2.20: For each pair of integers a, b and $3 \le a \le b$, there exist a connected graph G such that $g_1(G)=a$, $g_{E_0} = b$ and q=a+b.

Proof: Case(i) $a=b \ge 3$ Consider the graph wheel G= $K_{1,a}$, since G has exactly one vertex of degree 'a', $g_1(G)=a$ and by the Theorem 2.14, $g_{E_0} = a$. Hence a=b.



G Figure 2.4

Case(ii) 3<a<b. , let *G* be the graph obtained from the path on three vertices $P : u_1, u_2, u_3$ by adding $a - 2$ new vertices $v_1, v_2, \ldots, v_{a-2}$ and joining each $v_i (1 \le i \le a - 2)$ with u_1, u_2, u_3 and is shown in Figure 2.4. Also, since u_2 is the only full degree vertex of *G* by Theorem1.1, $g_1(G) = a - 2 + 3 - 1 = a$ and by Theorem 2.12, $g_{E_0} = b$ = q-a.

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